Game Theory

Part 2, Chapter 4

Roger Wattenhofer

Overview

- Selfish Caching
- Nash Equilibrium
- Price of Anarchy
- Rock Paper Scissor
- Mechanism Design

Selfish Peers

- Peers may not try to destroy the system, instead they may try to benefit from the system without contributing anything
- Such selfish behavior is called free riding or freeloading

- Free riding is a common problem in file sharing applications:
- Studies show that most users in the P2P file sharing networks do not want to provide anything

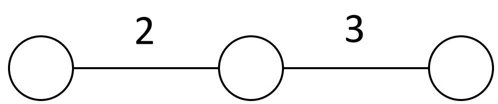
- Protocols that are supposed to be “incentive-compatible”, such as BitTorrent, can also be exploited
  - The BitThief client downloads without uploading!

Game Theory

- Game theory attempts to mathematically capture behavior in strategic situations (games), in which an individual's success in making choices depends on the choices of others.
- “Game theory is a sort of umbrella or 'unified field' theory for the rational side of social science, where 'social' is interpreted broadly, to include human as well as non-human players (computers, animals, plants)" [Aumann 1987]

Selfish Caching

- P2P system where node i experiences a demand w_i for a certain file.
  - Setting can be extended to multiple files
- A node can either
  - cache the file for cost α, or
  - get the file from the nearest node $l(i)$ that caches it for cost $w_i \cdot d_{i,l(i)}$
- Example: $\alpha = 4, w_i = 1$



What is the global „best“ configuration?
Who will cache the object?
Which configurations are „stable“?

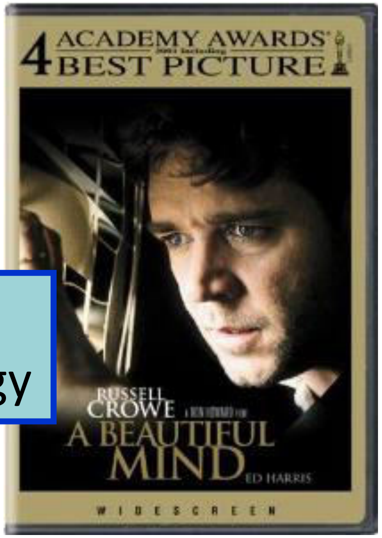
Social Optimum & Nash Equilibrium

- In game theory, the „best" configurations are called social optima
  - A social optimum maximizes the social welfare

Definition

A strategy profile is called social optimum iff it minimizes the sum of all cost.

  - A strategy profile is the set of strategies chosen by the players



- „Stable" configurations are called (Nash) Equilibria

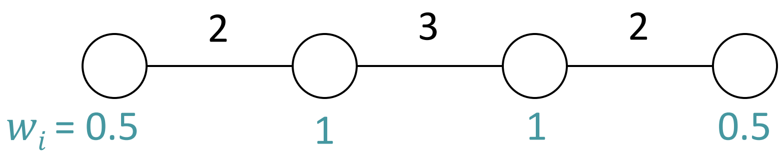
Definition

A Nash Equilibrium (NE) is a strategy profile for which nobody can improve by unilaterally changing its strategy

- Systems are assumed to magically converge towards a NE

Selfish Caching: Example 2

- Which are the social optima, and the Nash Equilibria in the following example?
  - $\alpha = 4$



- Nash Equilibrium $\not\Leftrightarrow$ Social optimum
- Does every game have
  - a social optimum?
  - a Nash equilibrium?

Selfish Caching: Equilibria

Theorem

Any instance of the selfish caching game has a Nash equilibrium

- **Proof by construction:**
  - The following procedure always finds a Nash equilibrium

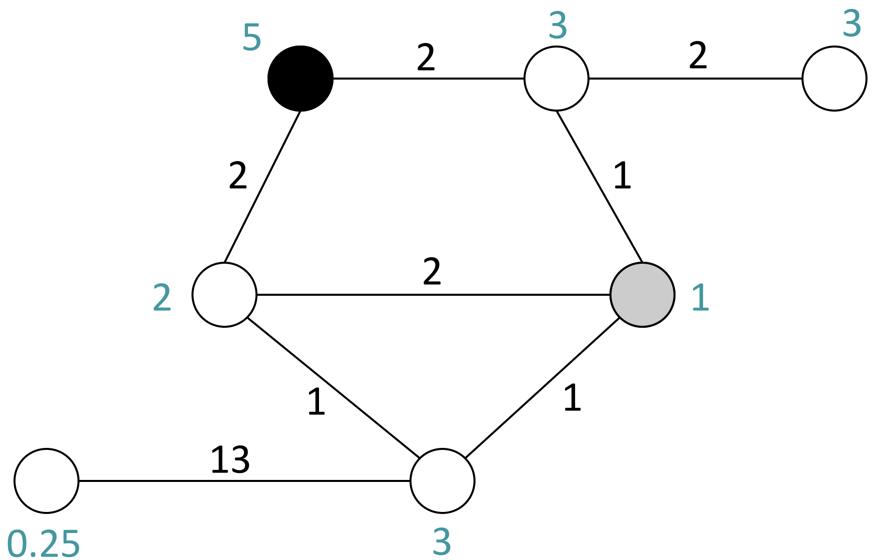
1. Put a node *y* with highest demand into caching set
2. Remove all nodes *z* for which $d_{zy} w_z < \alpha$
3. Repeat steps 1 and 2 until no nodes left

  - The strategy profile where all nodes in the caching set cache the file, and all others chose to access the file remotely, is a Nash equilibrium.

Selfish Caching: Proof example

1. Put a node *y* with highest demand into caching set
2. Remove all nodes *z* for which $d_{zy} w_z < \alpha$
3. Repeat steps 1 and 2 until no nodes left

$\alpha = 4$



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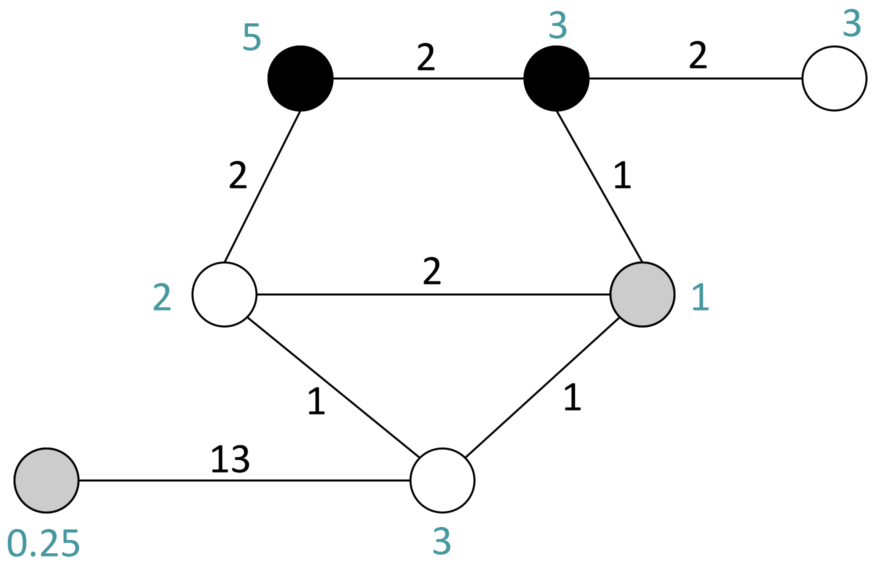

Selfish Caching: Proof example

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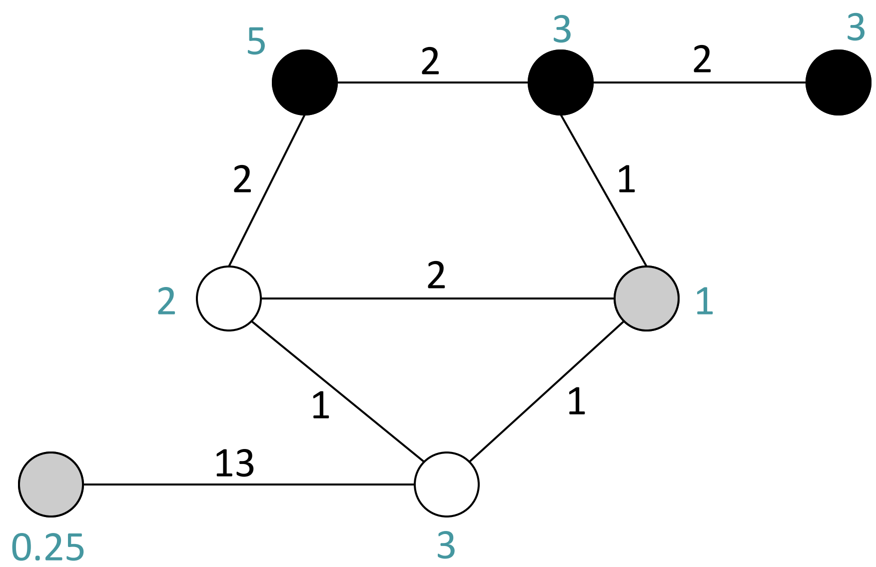
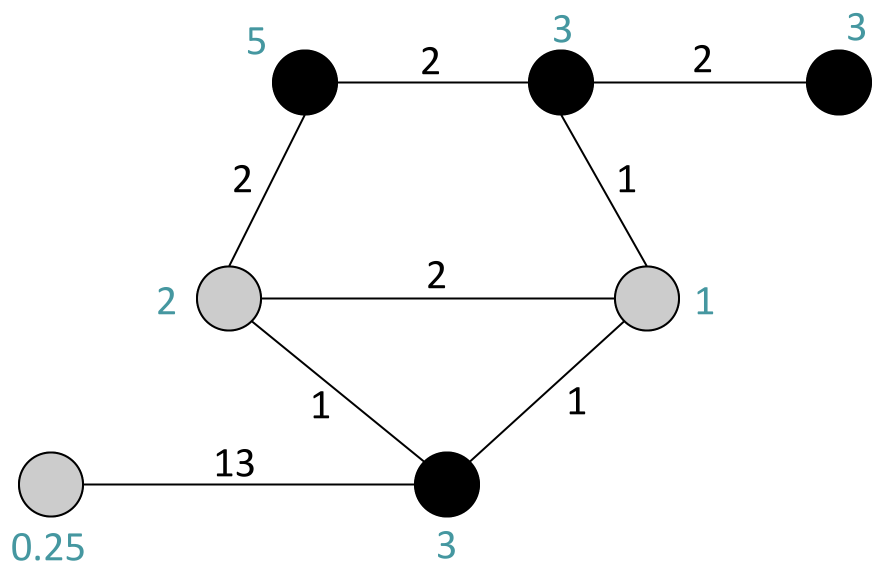


Selfish Caching: Proof example

1. Put a node *y* with highest demand into caching set
2. Remove all nodes *z* for which $d_{zy}w_z < \alpha$
3. Repeat steps 1 and 2 until no nodes left

$\alpha = 4$



– Does NE condition hold for every node?

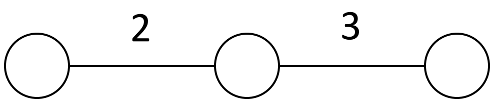
Proof

- If node x not in the caching set
  - Exists y for which $w_x d_{xy} < \alpha$
  - No incentive to cache because remote access cost $w_x d_{xy}$ are smaller than placement cost α

- If node x is in the caching set
  - For any other node y in the caching set:
    - Case 1: y was added to the caching set before x
      - It holds that $w_x d_{xy} \geq \alpha$ due to the construction
    - Case 2: y was added to the caching set after x
      - It holds that $w_x \geq w_y$, and $w_y d_{yx} \geq \alpha$ due to the construction
      - Therefore $w_x d_{xy} \geq w_y d_{yx} \geq \alpha$
  - x has no incentive to stop caching because all other caching nodes are too far away, i.e., the remote access cost are larger than α

Price of Anarchy (PoA)

- With selfish nodes any caching system converges to a stable equilibrium state
  - Unfortunately, NEs are often not optimal!



- Idea:
  - Quantify loss due to selfishness by comparing the performance of a system at Nash equilibrium to its optimal performance
    - Since a game can have more than one NE it makes sense to define a worst-case Price of Anarchy (PoA), and an optimistic Price of Anarchy (OPoA)

Definition

$$PoA = \frac{\text{cost}(worst\ NE)}{\text{cost}(social\ Opt)}$$

Definition

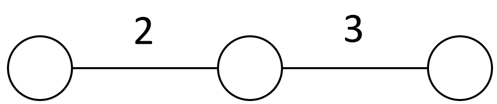
$$OPoA = \frac{\text{cost}(best\ NE)}{\text{cost}(social\ Opt)}$$

  - $PoA \geq OPoA \geq 1$
  - A PoA close to 1 indicates that a system is insusceptible to selfish behavior

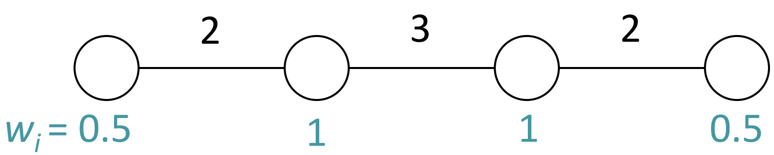
PoA for Selfish Caching

- How large is the (optimistic) price of anarchy in the following examples?

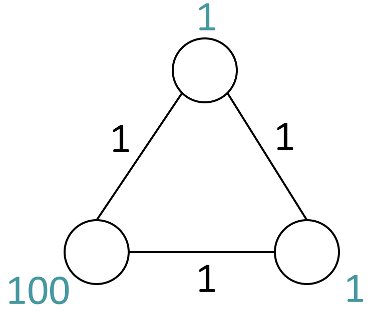
1) $\alpha = 4$, $w_i = 1$



2) $\alpha = 4$

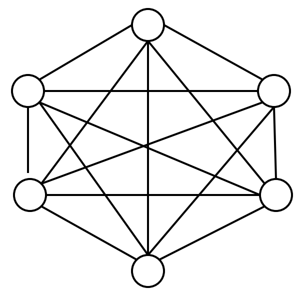


3) $\alpha = 101$

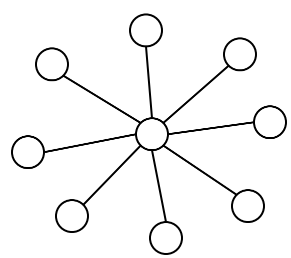


PoA for Selfish Caching with constant demand and distances

- PoA depends on demands, distances, and the topology
- If all demands and distances are equal (e.g. $w_i = 1, \;\; d_{ij} = 1$) ...
  - How large can the PoA grow in cliques?



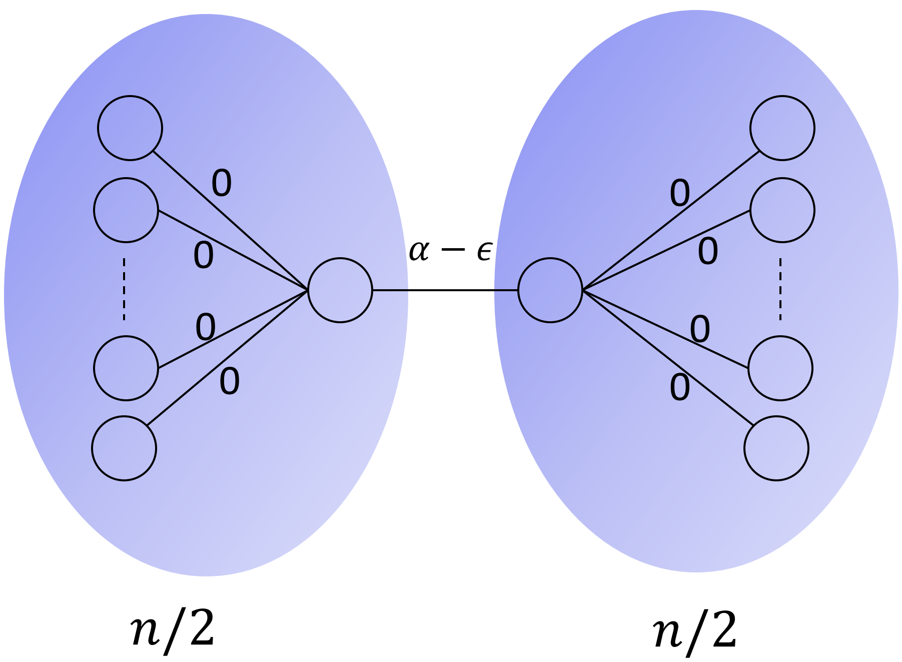
  - How large can the PoA grow on a star?



  - How large can PoA grow in an arbitrary topology?

PoA for Selfish Caching with constant demand

- PoA depends on demands, distances, and the topology
- Price of anarchy for selfish caching can be linear in the number of nodes even when all nodes have the same demand ($w_i = 1$)



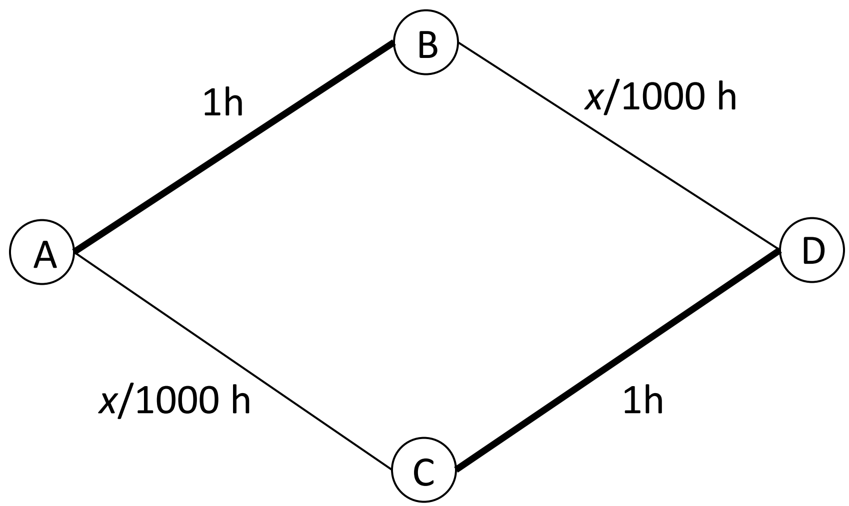
$$\text{cost}(NE) = \alpha + \frac{n}{2}(\alpha - \varepsilon)$$

$$\text{cost}(OPT) = 2 \cdot \alpha$$

$$PoA = OPoA \overset{\varepsilon \to 0}{=} \frac{1}{2} + \frac{n}{4} \in \Theta(n)$$

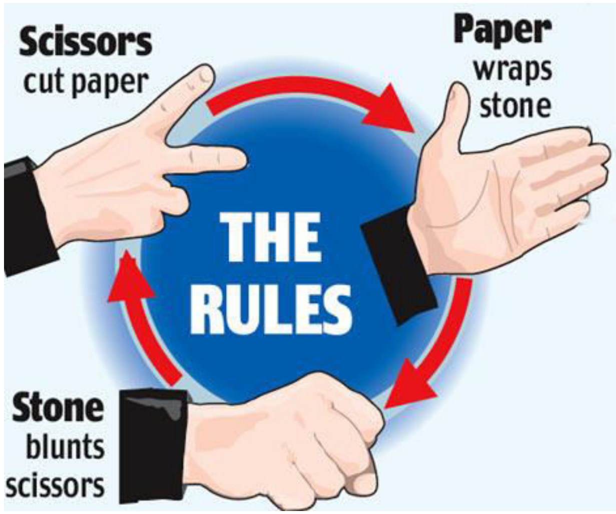
Another Example: Braess´ Paradox

- Flow of 1000 cars per hour from A to D
- Drivers decide on route based on current traffic
- Social Optimum? Nash Equilibrium? PoA?



- Is there always a Nash equilibrium?

Rock Paper Scissors



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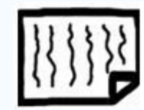
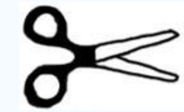
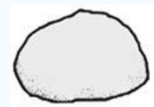
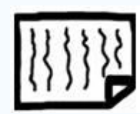
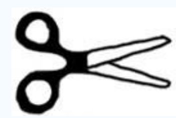
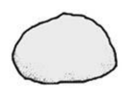
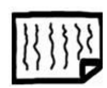
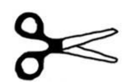
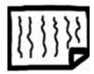
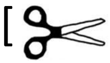
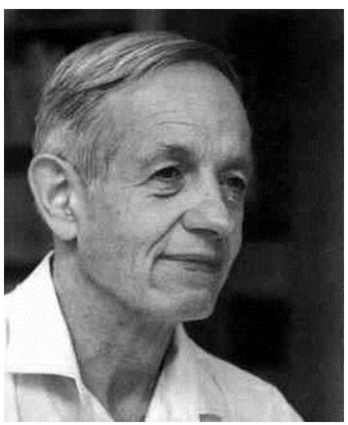
			
	0, 0	-1, 1	1, -1
	1, -1	0, 0	-1, 1
	-1, 1	1, -1	0, 0

- Which is the best action:  , , or  ?
- What is the social optimum? What is the Nash Equilibrium?
- Any good strategies?

Mixed Nash Equilibria

- Answer: Randomize !
  - Mix between pure strategies. A mixed strategy is a probability distribution over pure strategies.
  - Can you beat the following strategy in expectation?
    (p[] = 1/2, p[] = 1/4, p[] = 1/4)
  - The only (mixed) Nash Equilibrium is (1/3, 1/3, 1/3)
  - Rock Paper Scissors is a so-called Zero-sum game



Theorem [Nash 1950]

Every game has a mixed Nash equilibrium

Solution Concepts

- A solution concept predicts how a game turns out

Definition

A solution concept is a rule that maps games to a set of possible outcomes, or to a probability distribution over the outcomes

  - The Nash equilibrium as a solution concept predicts that any game ends up in a strategy profile where nobody can improve unilaterally.
    If a game has multiple NEs, then the game ends up in any of them.
- Other solution concepts:
  - Dominant strategies
    - A game ends up in any strategy profile where all players play a dominant strategy, given that the game has such a strategy profile
    - A strategy is dominant if, regardless of what any other players do, the strategy earns a player a larger payoff than any other strategy.
  - There are more, e.g. correlated equilibrium

Prisoner's Dilemma

- One of the most famous games in game theory is the so called Prisoner's Dilemma
  - Two criminals A and B are charged with a crime, but only circumstantial evidence exists
  - Both can cooperate (**C**), i.e., stay silent or they can defect (**D**), i.e., talk to the police and admit their crime
  - If both cooperate, each of them has to go to prison for one year
  - If both defect, each of them has to go to prison for three years
  - If only A defects but B chooses to cooperate, A is a crown witness and does not have to serve jail time but B gets three years (and vice versa)

	C	**D**
C	-1, -1	-3, 0
D	0, -3	-2, -2

- Dominant strategy is to defect

How can Game Theory help?

- Economy
  - Understand markets?
  - Predict economy crashes?
  - Sveriges Riksbank Prize in Economics (“Nobel Prize”) has been awarded many times to game theorists

- Problems
  - GT models the real world inaccurately
  - Many real world problems are too complex to capture by a game
  - Human beings are not really rational

- GT in computer science
  - Players are not exactly human
  - Explain unexpected deficiencies (emule, bittorrent etc.)
  - Additional measurement tool to evaluate distributed systems

Mechanism Design

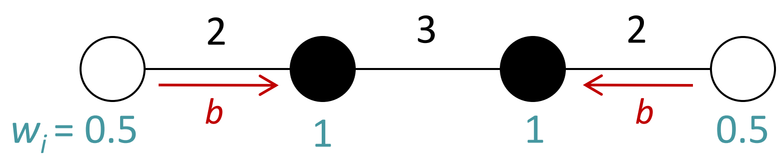
- Game Theory describes existing systems
  - Explains, or predicts behavior through solution concepts (e.g. Nash Equilibrium)
- Mechanism Design creates games in which it is best for an agent to behave as desired by the designer
  - incentive compatible systems
  - Most popular solution concept: dominant strategies
  - Sometimes Nash equilibrium
  - Natural design goals
    - Maximize social welfare
    - Maximize system perfomance

Mechanism design ≈ „inverse“ game theory

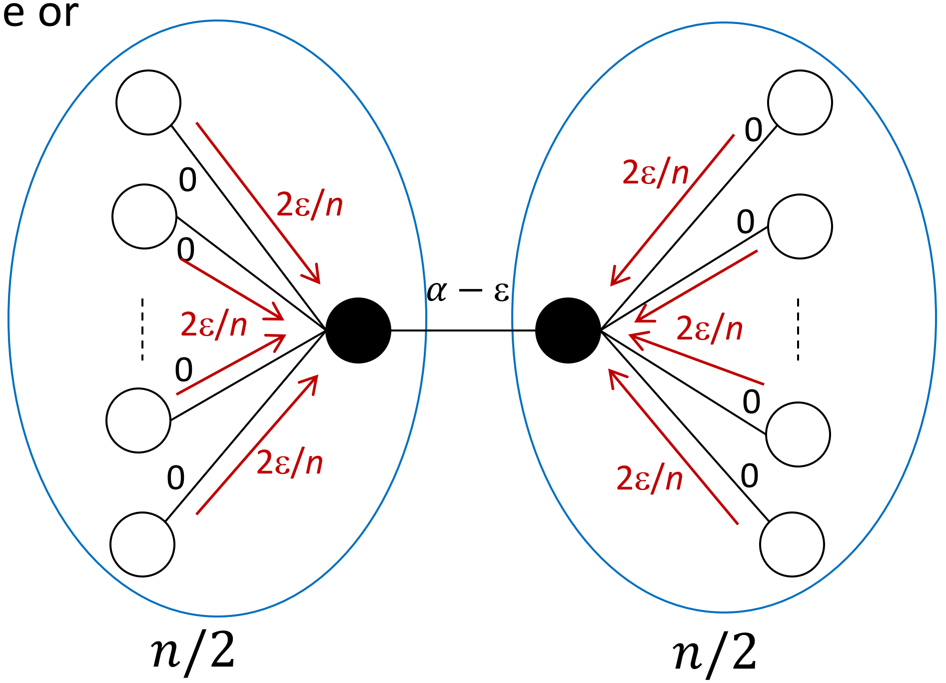
Incentives

- How can a mechanism designer change the incentive structure?
  - Offer rewards, or punishments for certain actions
    - Money, better QoS
    - Emprisonment, fines, worse QoS
  - Change the options available to the players
    - Example: fair cake sharing (MD for parents)

Selfish Caching with Payments

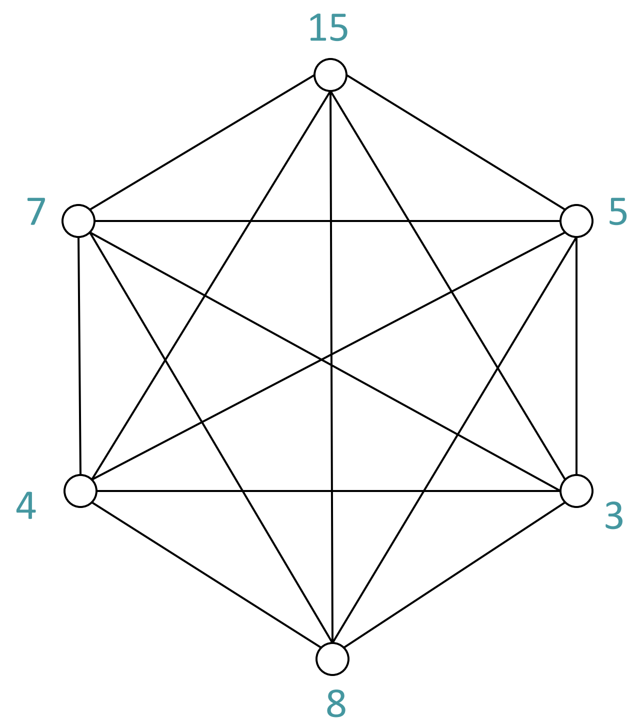


- Designer enables nodes to reward each other with payments
- Nodes offer bids to other nodes for caching
  - Nodes decide whether to cache or not after all bids are made
- $OPoA = 1$
- However, PoA at least as bad as in the basic game



Selfish Caching: Volunteer Dilemma

- Clique
  - Constant distances $d_{ij} = 1$
  - Variable demands $1 < w_i < \alpha = 20$
- Who goes first?
  - Node with highest demand?
  - How does the situation change if the demands are not public knowledge, and nodes can lie when announcing their demand?

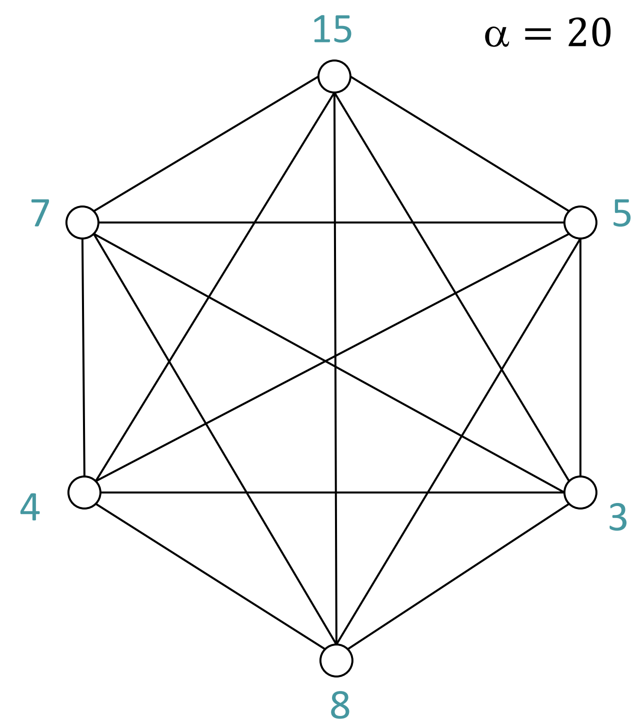


First-Price Auction

- Mechanism Designer
  - Wants to minimize social cost
  - Is willing to pay money for a good solution
  - Does not know demands w_i

Idea: Hold an auction

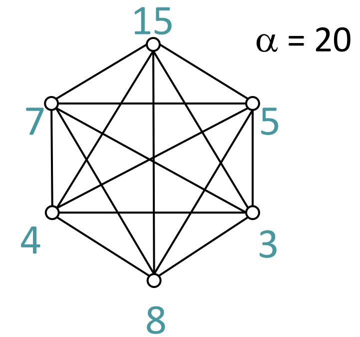
  - Auction should generate competition among nodes. Thus get a good deal.
  - Nodes place private bids b_i. A bid b_i represents the minimal payment for which node i is willing to cache.
  - Auctioneer accepts lowest offer.
    Pays $b_{min} = \min_i b_i$ to the bidder of b_{min}.

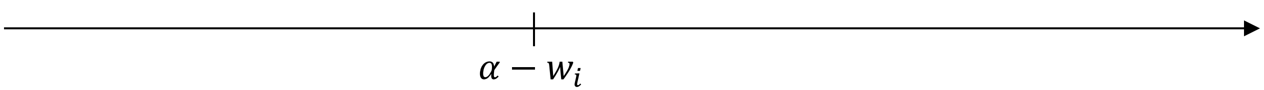


- What should node *i* bid?
  - $\alpha - w_i \leq b_i$
  - i does not know other nodes‘ bids

Second-Price Auction

- The auctioneer chooses the node with the lowest offer, but pays the price of the second lowest bid!
- What should i bid?
  - Truthful ($b_i = \alpha - w_i$), overbid, or underbid?





Theorem

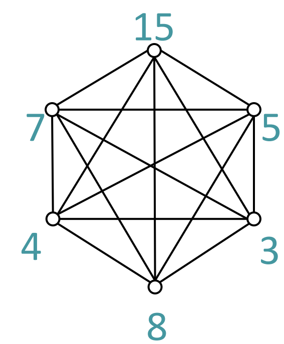
Truthful bidding is the dominant strategy in a second-price auction

Proof

- Let $v_i = \alpha - w_i$. Let $b_{min} = \min_{j \neq i} b_j$.
- The payoff for i is $b_{min} - v_i$ if $b_i < b_{min}$, and 0 otherwise.
- „truthful dominates underbidding“
  - If $b_{min} > v_i$ then both strategies win, and yield the same payoff.
  - If $b_{min} < b_i$ then both strategies lose.
  - If $b_i < b_{min} < v_i$ then underbidding wins the auction, but the payoff is negative. Truthful bidding loses, and yields a payoff of 0.
  - Truthful bidding is never worse, but in some cases better than underbidding.
- „truthful dominates overbidding“
  - If $b_{min} > b_i$ then both strategies win and yield the same payoff
  - If $b_{min} < v_i$ then both strategies lose.
  - If $v_i < b_{min} < b_i$ then truthful bidding wins, and yields a positive payoff. Overbidding loses, and yields a payoff of 0.
  - Truthful bidding is never worse, but in some cases better than overbidding.
- Hence truthful bidding is the dominant strategy for all nodes i.

Another Approach: 0-implementation

- A third party can implement a strategy profile by offering high enough „insurances“
  - A mechanism implements a strategy profile S if it makes all strategies in S dominant.



- Mechanism Designer publicly offers the following deal to all nodes except to the one with highest demand, p_{max}:
  - „If nobody choses to cache I will pay you a millinillion.“
- Assuming that a millinillion compensates for not being able to access the file, how does the game turn out?

Theorem

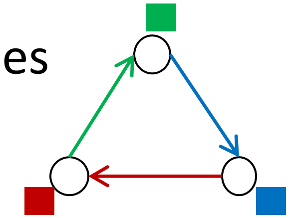
Any Nash equilibrium can be implemented for free

MD for P2P file sharing

- Gnutella, Napster etc. allow easy free-riding
- BitTorrent suggests that peers offer better QoS (upload speed) to collaborative peers
  - However, it can also be exploited
  - The BitThief client downloads without uploading!
    - Always claims to have nothing to trade yet
    - Connects to much more peers than usual clients

- Many techniques have been proposed to limit free riding behavior
  - Tit-for-tat (T4T) trading
    - Allowed fast set (seed capital),
    - Source coding,
    - indirect trading,
    - virtual currency…

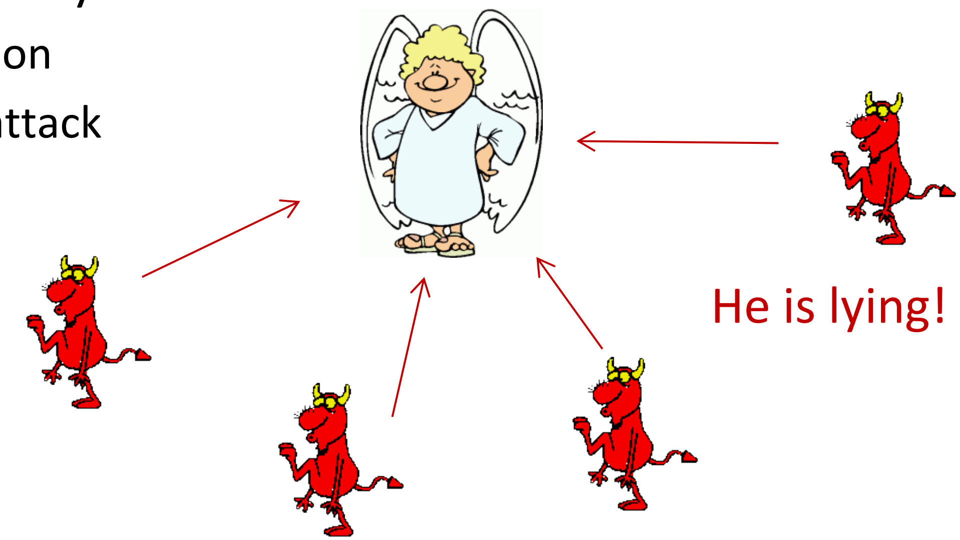
    } increase trading opportunities



  - Reputation systems
    - shared history

MD in Distributed Systems: Problems

- Virtual currency
  - no trusted mediator
  - Distributed mediator hard to implement
- Reputation systems
  - collusion
  - Sibyl attack



- Malicious players
  - Nodes are not only selfish
    but sometimes Byzantine

Credits

- The concept for a Nash Equilibrium is from John Nash, 1950
- The definition of a Price of Anarchy is from Koutsoupias and Papadimitriou, 1999
- The Selfish Caching Game is from Chun, Chaudhuri, Wee, Barreno, Papadimitriou, and Kubiatowicz, 2004
- The Prisoner’s Dilemma was first introduced by Flood and Dresher, 1950
- A generalized version of the second-price auction is a VCG auction, introduced by Vickrey, Clarke, and Groves, 1973

That's all, folks!

Questions & Comments?

Roger Wattenhofer